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Abelian gauge fields coupled to simplicial quantum gravity

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Abstract: We study the coupling of abelian gauge theories to four-dimensional simplicial quantum gravity. The gauge fields live on dual links. This is the correct formulation if we want to compare the effect of gauge fields on geometry with similar effects studied so far for scalar fields. It shows that gauge fields couple equally weakly to geometry as scalar fields, and it offers an understanding of the relation between measure factors and abelian gauge fields observed so-far.

Keywords: Models of Quantum Gravity, 2D Gravity, Lattice Models of Gravity.

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Contents

1. Introduction

A path integral formulation of quantum gravity includes in its simplest version an integration over all four-dimensional geometries of a given fixed topology, the weight of each geometry being the Boltzmann weight of a suitable action. An attempt to implement such a path integral representation is known as *dynamical triangulation*. The integration over geometries is approximated by a summation over all triangulations constructed from equilateral simplices of side-length a, and the continuum limit is obtained by letting $a \to 0$ for a suitable choice of bare coupling constants in the action [1, 2].

This model has been analyzed in a series of papers, both with and without coupling to scalar fields. Originally the hope was that an observed second order phase transition could be used to define a non-perturbative theory of quantum gravity, but a closer examination revealed that the transition was (weakly) first order [3]. Motivated by an effective theory of quantum gravity, which showed that the gauge fields coupled very strongly to the infrared sector of gravity [4], the coupling between abelian gauge fields and simplicial quantum gravity was introduced and studied using the non-compact version of the gauge field [5]. We shall refer to this version of the model as AGM (abelian gauge model). Indeed, for the first time a significant coupling between matter and gravity was observed, and a new phase, named the "crinkled" phase was observed. It appeared to be different from the so-called "crumpled" and "elongated" phases observed that far, and which both seemed irrelevant for a continuum limit of quantum gravity. The crinkled phase had a negative entropy exponent γ and a fractal dimension $d_H \approx 4$. However, it was later shown that most of the properties of this new phase could be obtained by a simple change in the measure of the path integral [6], and since such a change is of ultra-local nature it is unlikely to provide a faithful representation of the ideas relating the trace anomaly to the infrared behaviour of quantum gravity, assuming conformal invariance.

While the change in measure seems to capture most of the gravitational physics associated with the coupling of abelian matter fields to simplicial quantum gravity, the two theories are of course not equivalent, as emphasized in [5, 6]. One can explore the full consequences of the abelian gauge fields coupled to simplicial quantum gravity by Monte Carlo simulations. Such studies were performed in [5] and few ambiguities as to the nature of the transition to the crinkled phase and of the crinkled phase itself remained. For this reason and as a matter of principle it is valuable to have an independent simulation of the system.

In the non-compact formulation the abelian gauge action is gaussian. There exist well-known methods relating low-dimensional gaussian theories on a direct lattice to their *dual* versions. The methods developed on regular lattices work equally well on dynamical lattices and we shall make use of the concept of duality to relate theories defined on direct and dual lattices. This relation will highlight why the originally chosen way of coupling gauge fields to geometry produced results almost identical to a simple measure term. It will show that this is the result of a somewhat unfortunate choice of action, and that the "real" abelian degrees of freedom couple to simplicial quantum gravity more or less as ordinary massless scalar fields, i.e. they do not influence the phase structure of pure simplicial quantum gravity, at least for the number of independent abelian fields used so far. From this point of view the crinkled phase observed so far is *entirely* due to the insertion of an ultra-local measure factor. Further, the way we implement the coupling between the gauge fields and geometry allows us to introduce new effective actions which might change the phase diagram of matter coupled to gravity.

Another possibility, not studied in this paper would be to go to the compact formulation of the gauge action. In this case, at least in principle, one may expect a phase transition between the confined and deconfined phases. Numerical results for the three-dimensional case suggest however that such a transition disappears, at least for a single U(1) field [9]. Whether some new physics may emerge for more fields or for the non-abelian gauge fields remains an open problem.

2. The model

Let T be an abstract triangulation of a four-dimensional manifold. Let T^* be the simplicial complex dual to T and N_d denote the number of d-dimensional sub-simplices in T. N_4 is the total volume of the manifold. We shall compare two possible methods of introducing the (non compact) abelian gauge fields on such a manifold.

The first possibility, which we call the T implementation, would be to put the (complex) matter fields at the vertices in T and the U(1) gauge potentials $A_{IJ} = -A_{JI}$ on the (oriented) links (IJ) of the triangulation T, with I, J numbering the vertices of the triangulation. At each vertex I we can perform the gauge transformation χ_I and the gauge fields transform as

$$A_{IJ} \to A_{IJ} + \chi_I - \chi_J \,. \tag{2.1}$$

Let us now choose a spanning tree in the complex T, i.e. a connected graph of links with no loops. The number of links in the spanning tree is $N_0 - 1$. Making use of the gauge transformation (2.1) we can perform the gauge fixing of all the gauge potentials on the spanning tree, reducing them to zero. The resulting number of degrees of freedom is $N_1 - N_0 + 1$. In principle there is nothing wrong with this implementation where the matter fields live on the vertices. However, the number of the degrees of freedom both of the matter fields and the gauge fields has no simple relation to the volume of the manifold. For the scalar fields this number is proportional to the number of vertices N_0 (and not N_4), which can grow as N_4^{α}, where $\alpha \leq 1$. In cases where one has a sensible thermodynamic limit it should not matter which way the coupling between matter and gravity is done, but since "pathological" triangulations can occur, and even dominate for some choices of gravitational coupling constants, this choice of implementation may have troubles with defining the extensive quantities, like the average action etc.

The alternative possibility, which we call the T^* implementation, is to choose the U(1) gauge potentials A^*_{ij} to live on the oriented links l^*_{ij} of the dual graphs, connecting simplices with labels i and j. Each dual link corresponds to an interface between the neighbouring simplices. As usual $A^*_{ji} = -A^*_{ij}$ and there are $N_3 = \frac{5}{2}N_4$ such fields. This formulation is in accordance with most simulations of matter fields coupled to gravity. The matter fields are usually associated with the four-simplices (are located in the "centers" of the four-simplices) and their derivatives are calculated as differences formed between the values of the fields in neighbouring four-simplices, i.e. they are associated with the dual links. The reason for choosing such an implementation, apart from being convenient in numerical simulations, is that the naive counting of field degrees of freedom will be proportional to the four-volume N_4. This property holds also for the gauge degrees of freedom. At each simplex we can perform a gauge transformation χ^*_i and the gauge fields transform as

$$A^*_{ij} \to A^*_{ij} + \chi^*_i - \chi^*_j \,. \tag{2.2}$$

Let us now choose a spanning tree in the complex T^*. The number of links in the spanning tree is $N_4 - 1$. One can gauge $A^*_{ij} = 0$ for all links in the spanning tree. This leaves us with $N_3 - N_4 + 1 = \frac{3}{2}N_4 + 1$ physical degrees of freedom for the gauge field. The nice feature is that this number depends only on the volume N_4.

In both implementations the independent degrees of freedom can be parametrized by the gauge invariant (oriented) plaquette variables. In the T implementation these are oriented triangles t_{IJK} and the plaquette variables satisfy:

$$P(t_{IJK}) = A_{IJ} + A_{JK} + A_{KI}\,. \tag{2.3}$$

The plaquettes are not independent variables. Because of the abelian nature of the gauge fields $P(t_{IJK})$ are unchanged under the cyclic permutation of I, J, K and change sign when the orientation of the plaquette is inversed. For each triangulation one has to define the positive orientation for the plaquette variables to avoid the double counting. By construction the flux through any closed two-dimensional surface is zero. The number of independent such surfaces is (in case the topology is that of the 4-sphere, which we will assume)

$$N(S) = N_3 - N_4 + 1. \tag{2.4}$$

This is the number of independent Bianchi identities we can write down. By Euler's relation for the 4-sphere it leaves precisely $N_1 - N_0 + 1$ independent plaquettes, and the jacobian for changing from independent gauge fields to plaquette variables is one [10]:

$$\int \prod_{l\in T} \mathrm{d}A_l \prod_{l'\in ST} \delta(A_{l'})\; F(P(A)) = \int \prod_{t} \mathrm{d}P(t) \prod_{s\in S} \delta(\sum_{t\in s} P(t))\; F(P). \tag{2.5}$$

In eq. (2.5) ST denotes a spanning tree in T and S denotes the set of $N(S)$ independent two-dimensional surfaces where the Bianchi identities are enforced. l and t are respectively links and triangles of the manifold and we assume that the definition of the positive orientation of links and triangles was uniquely chosen. Notice that in this implementation each elementary gauge loop (plaquette) has a length three, however each gauge potential contributes to a number of neighbouring triangles which is a non-trivial local geometric characteristic of the system.

Similar discussion can be made in the T^* implementation. The dual plaquettes constructed from a D-dimensional triangulation can be labelled by the $(D-2)$-dimensional sub-simplices in the D-dimensional triangulation, which are "encircled" by the associated dual plaquette. In four dimensions we shall label dual plaquettes by the triangles they encircle. The geometric properties of the two gauge-invariant objects $P(t_i)$ and $P^*(t_i)$ are however different: The length of the plaquette $P^*(t_i)$ is now $o(t_i)$, where $o(t_i)$ is the order of the triangle t_i, or the number of four-dimensional simplices sharing this triangle. As before we have to choose a positive orientation, and the plaquette variable is unchanged when the corresponding loop is rotated and changes sign, when it is inverted. Each gauge potential A^*_{ij} contributes to exactly four dual plaquette variables. Like in the T implementation we can parametrize the system by the plaquette variables $P^*(t_i)$, satisfying a set of the Bianchi identities.

To this point we did not specify the model we want to consider. Let us start with the T^* implementation. The plaquette variables are the sums of gauge fields associated with the oriented (dual) links 'encircling' the triangles t_i in T:

$$P^*(t_i) = \sum_\alpha A^*(l^*_\alpha)\,, \tag{2.6}$$

where the orientation of the (dual) links follows that of the plaquette and as discussed above $t_i \in T$ can be viewed as dual to the plaquette. To keep the discussion general let us at this point assume only that the gauge action is gaussian and assumes a form:

$$S^*_g(T, A^*) = \sum_i \beta_i P^{*2}(t_i)\,, \tag{2.7}$$

where β_i are positive, local, geometry-dependent coefficients. We shall argue below what seems to be the most natural choice of β_i.

With this notation the partition function can be written as

$$Z = \sum_T \frac{1}{C_T} \mathrm{e}^{-\kappa_4^* N_4(T) + \kappa_2^* N_2(T)} \int \prod_{l^*} \mathrm{d}A^*_{l^*} \prod_{l'^* \in ST^*} \delta(A_{l'^*})\ \mathrm{e}^{-S^*_g(T,A^*)}\,, \tag{2.8}$$

where ST^* is a spanning tree in T^*.

We shall now construct the dual version of this model. We can write

$$\exp(-\beta_i P^{*2}(t_i)) = \sqrt{\frac{1}{\pi\beta_i}} \int dp(t_i)\ \exp\Big(-\frac{p^2(t_i)}{\beta_i} + 2ip(t_i)P^*(t_i)\Big)\,. \tag{2.9}$$

Using this integral representation of the plaquette, as well as the standard integral of the δ-function

$$\prod_{l' \in ST^*} \delta(A^*_{l'}) = \int \prod_{l' \in ST^*} \frac{\mathrm{d}\alpha_{l'}\ \mathrm{e}^{2i\alpha_{l'} A^*_{l'}}}{\pi}\,, \tag{2.10}$$

the integration over the A^*-fields in T^* can be performed, giving rise to a set of delta functions:

$$\prod_{l^*} \pi\delta\left(\alpha_{l^*} + \sum_{t_i \ni l^*} p(t_i)\right), \tag{2.11}$$

where the product is over the dual links l^*, and $\alpha_{l^*} = 0$ if the dual link l^* does not belong to the chosen spanning tree. Notice that the sum in the argument of each δ function has exactly four terms. Performing the α_{l^*} integration in (2.10) eliminates $N_4(T) - 1$ of the $N_3(T)$ δ-functions in (2.11). The remaining $N_3(T) - N_4(T) + 1$ δ-functions can be viewed as the Bianchi identities for the $N_2(T)$ variables $p(t_i)$. Since there are $N_2(T) - (N_3(T) - (N_4(T) - 1))$ independent variables $p(t_i)$, we can implement the Bianchi identities by introducing U(1) gauge fields A_l living on the links of the triangulation T, rewriting $p(t_i) = P(t_i) = \sum_{l \in t} A_l$. Independent gauge transformations can be done at vertices, and after gauge fixing we are left with $N_1(T) - (N_0(T) - 1)$ independent A_l fields, i.e. precisely the same as the number of independent $p(t_i)$ fields.

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We end up this discussion by writing down the identity valid for each triangulation:

$$\int \prod_{l^*} \mathrm{d}A^*_{l^*} \prod_{l'^* \in ST^*} \delta(A_{l'^*})\, \mathrm{e}^{-S^*_g(T,A^*)} = \pi^{-\frac{1}{2}N_2 + \frac{3}{2}N_4 + 1}\, \mathrm{e}^{-\frac{1}{2}\sum_{t_i} \ln \beta_i} \times \\ \times \int \prod_l \mathrm{d}A_l \prod_{l' \in ST} \delta(A_{l'})\, \mathrm{e}^{-S_g(T,A)}, \tag{2.12}$$

where

$$S_g(T,A) = \sum_{t_i} \frac{P^2(t_i)}{\beta_i} \tag{2.13}$$

and $P(t_i)$ is the oriented plaquette variable (2.3) expressed in terms of the fields A_l living on the links of the triangulation T. For the partition function (2.8) we get

$$Z = \sum_T \frac{\pi}{C_T} \mathrm{e}^{-\kappa_4 N_4(T) + \kappa_2 N_2(T) - \frac{1}{2}\sum \ln \beta_i} \int \prod_l \mathrm{d}A_l \prod_{l' \in ST} \delta(A_{l'})\, \mathrm{e}^{-S_g(T,A)}, \tag{2.14}$$

where ST is a spanning tree in T and

$$\kappa_4 = \kappa_4^* - \frac{3}{2} \ln \pi, \qquad \text{and} \qquad \kappa_2 = \kappa_2^* - \frac{1}{2} \ln \pi. \tag{2.15}$$

Thus, apart from a shift $\kappa_4 \to \kappa_4^* - \frac{3}{2}\ln\pi$ and $\kappa_2 \to \kappa_2^* - \frac{1}{2}\ln\pi$, the two models are dual up to the weight factor $-\frac{1}{2}\sum \ln \beta_i$.

The choice of the action parameters β_i should be made at this point. One can argue that it is natural to choose $\beta_i = 1/o(t_i)$, where $o(t_i)$ is the order of the triangle. In the T^* implementation $o(t_i)$ is also the length of the boundary of the plaquette dual to t_i, and, assuming the plaquette to be flat, also proportional to it's area. Since $P^*(t_i)$ signifies the flux of the plaquette dual to t_i, the field strength is proportional to $P^*(t_i)/o(t_i)$, while the volume element associated with t_i likewise is proportional to $o(t_i)$. This argument assumes that the plaquette is a flat two-dimensional object with a constant field strength. This needs not be the case, so another possible choice could be $\beta_i = 1/o^\alpha(t_i)$ with some power α different from 1, which leaves space for some non-trivial geometry of plaquettes.

The model for a single non-compact abelian gauge field can readily be generalized to n_g copies. For a given triangulation, T, these models will be non-interacting, but summing over all triangulations one introduces interactions between the copies, mediated by the geometry, as well as interactions between geometry and the matter fields. The final action we use can thus be written as

$$S^*_g = \sum_{i,k} \frac{P^{*2}(t_i,k)}{o^\alpha(t_i)}, \tag{2.16}$$

where the index k numbers the copies of abelian gauge fields and takes values

$1,\ldots,n_g$. If $\alpha = 1$ the model is dual to the original AGM model except for a measure factor

$$\frac{1}{2} n_g \sum_t \ln o(t) \tag{2.17}$$

for each triangulation and a shift in the gravitational and cosmological constants by

$$\kappa_4 \to \kappa_4 + \frac{3n_g}{2} \ln \pi \quad \text{and} \quad \kappa_2 \to \kappa_2 + \frac{n_g}{2} \ln \pi \,. \tag{2.18}$$

We call this version of the model the dual abelian gauge model (DAGM).

We can make two remarks at this point:

- The whole derivation made explicitly use of the fact that the topology of the manifold is that of a four-sphere. For different topologies the duality relations become slightly more complicated and new degrees of freedom appear, related to the possible topologically non-trivial boundary conditions for the gauge potentials.

- Similar discussion can be made in the three and the two-dimensional cases. In three dimensions in the T^* implementation the dual of a plaquette is a link. The duality relates the gauge theory on dual plaquettes to a massless bosonic theory, where the bosonic field live on vertices, again up to a ultra-local measure term. The T implementation will have it's dual in the form of a massless scalar field living in the centers of the simplices. In two dimensions the duality transformation permits to integrate out the gauge field completely, leaving only the measure term.

3. The algorithm

The algorithm describing numerical simulations of the four-dimensional simplicial gravity was described in many papers. Here we shall not repeat details concerning the five geometric "moves" which became a standard. There are however some peculiarities of the present simulation, which make the updating in the gauge sector rather non-trivial.

In order to perform a Monte Carlo simulation of the model we found it convenient to work directly with the gauge invariant plaquettes rather than the gauge fields themselves. Occasionally we shall need the gauge fields and we shall then reconstruct them from the plaquette variables. This is necessary only in order to control round-off errors, which eventually will lead to a violation of the Bianchi identities. The geometric "moves" create and destroy simplices. In effect also the new gauge potentials, located at the interfaces between simplices are created and destroyed. Since the gauge action is gaussian we shall use the heat bath method to generate the new variables. In order to find the detailed balance condition it is always necessary to compare the "move" and it's inverse.

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The addition of the new gauge potentials does not create any problems and can be handled in a way closely resembling that of the gaussian scalar fields. The procedure has two steps: in the first step we decide if the move will be performed. If the answer is yes we proceed to the second step, where the new gauge potentials are generated from a (multi-dimensional) normal distribution. In the process the old plaquette variables are modified and new plaquette variables appear. The gaussian form of the gauge potential guarantees that both the modified and new plaquettes will remain restricted in value. The acceptance probability for the first step can be expressed in terms of the gauge invariant plaquette variables. In the second step the values of new gauge potentials are generated, but only the gauge invariant information (the plaquette variables) is stored.

Situation may be quite different when we try the inverse operation deleting some gauge potentials. If we would decide to store the values of the potentials and simply subtract these values from plaquettes, the resulting change in the gauge action could become arbitrarily large. The reason is that only the gauge invariant combinations of the gauge potentials are physical (and in the Monte Carlo simulation restricted by the gauge action), while the gauge potentials themselves may become large unless some form of the gauge fixing is imposed. Introducing the gauge fixing (a spanning tree) is however a global problem and therefore maintaining the gauge condition in the numerical simulation, where the geometry is dynamical would be very costly and impractical. We have developed a different approach, which is again a modification of the heat bath algorithm. In the first step we decide if the inverse move will be performed. In order to find the acceptance we integrate over all possible gauge choices of the gauge potentials to be deleted. The resulting formula is local and expressible in terms of the gauge invariant quantities. If the inverse move is accepted we proceed to the second step, where the potentials to be deleted are generated from the appropriate normal distribution. After the inverse move only the gauge invariant information is stored.

In the program the situation is slightly more complicated. In order to maintain the detailed balance at each move we have to consider the acceptance for both the move and it's inverse. Also the geometric moves used in the four dimensional updating of geometry in general are rather complicated and in each move some gauge potentials are created, while other are destroyed. We shall report more details about the updating procedure in the appendix.

4. Observables and measurements

Basic observables in simplicial quantum gravity have already been discussed in a number of articles. Since we are here interested in gauge fields it would be natural to measure gauge invariant observables. An obvious gauge invariant quantity is the correlator $\left\langle F^2_{\mu\nu}(r)F^2_{\lambda\rho}(0)\right\rangle$, where the average is over all geometries with S^4 topology and with points separated a geodesic distance r. The gauge field itself is just gaussian, so the propagator in flat space is trivially calculated. In quantum space-time, where we average over all geometries , it is not clear precisely how the propagator will fall off with the geodesic distance. In fact, the motivation for this study was to test if the gauge fields had a stronger interaction with geometry than the scalar fields investigated so far. In [17] it was shown how to extract the *connected* correlator for matter fields coupled to simplicial quantum gravity. However, in order to obtain a non-trivial result of interest for continuum physics it is first necessary to identify a point in the combined coupling constant space of gravity and gauge theory where we can take an interesting continuum limit. In the following we concentrate on this necessary first step. Thus we study the κ_2, κ_4 phase diagram of the theory for different values of the parameter α in eq. (2.16). For a given value of κ_2 there exists a critical point κ_4 where an infinite volume limit can be obtained. At this critical point one can measure geometric quantities like the entropy exponent γ, which describes the distribution of baby universes, and one can measure the fractal dimension of space-time. As is standard in the measurements of the fractal dimension in simplicial quantum gravity studies, one operates with two, in principle independent, definitions of the fractal dimension. A "short distance" fractal dimension, d_h, measures how the volume $V(r)$ of spherical balls of geodesic radius r grows with increasing r as long as r is much smaller than the average radius R of the universe:

$$V(r) \sim r^{d_h}\,, \qquad \text{for} \quad r \ll R\,. \tag{4.1}$$

A "cosmological" fractal dimension is defined by the way "macroscopic" distances scale with the cosmological constant or with the average space-time volume. For instance

$$V(r) \sim r^{d_h - b} F\Big(\frac{r}{V^{1/d_H}}\Big)\,, \tag{4.2}$$

where $F(x) \sim x^b$ for $x \to 0$. In the four-dimensional gravity models studied so far one has always found that $d_h = d_H$. Although there exist models [14] of fractal structures where $d_h \neq d_H$, one expects that $d_H = d_h$ if a sensible thermodynamic limit exists [12, 13].

Since the transition between the branched polymer phase and the crumpled phase of four dimensional gravity is characterized by the appearance of singular vertices of very high order, we also study the distribution of the order of vertices, links, and triangles in order to characterize the various phases we observe.

4.1 The case $\alpha = 1$

Since the model (2.16) for $\alpha = 1$ is dual to the original AGM model if we include the weight factor (2.17) and make the shift (2.18), we can as a calibration reproduce the κ_2, κ_4 diagram of AGM. This is shown in fig. 1 for the number of gauge fields $n_g = 3$.

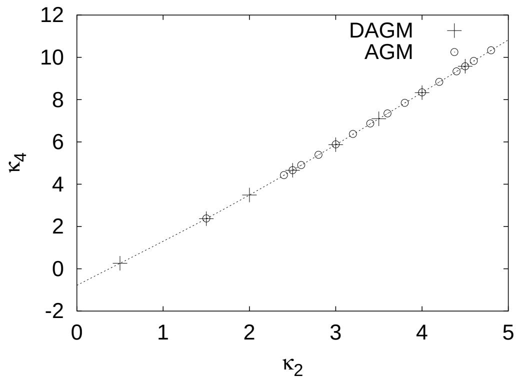

Figure 1: The (κ_2, κ_4) diagram of the AGM and that of the dual model (DAGM) studied in this paper. Error bars are smaller than the point size and the line is only to guide the eye. $n_g = 3$ and $N_4 = 4000$.

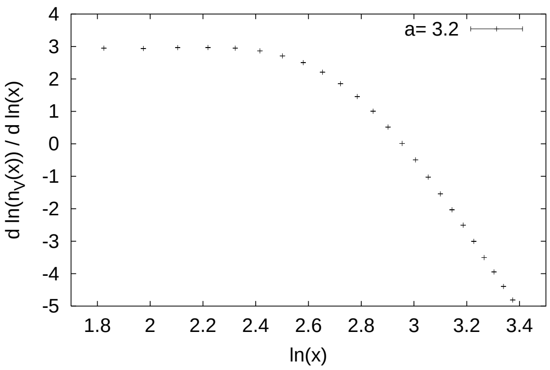

Figure 2: The logarithmic derivative of the two point function $n_V(r)$. Here $x = (r + a)/V^{1/d_h}$ where $V \equiv N_4$. For a suitable choice of the shift a a power law defining d_h can be obtained.

We see perfect agreement. As a general statement we also see approximately the same values of γ and d_h as observed in the AGM model, thus verifying the results obtained in these investigations. It is illustrated in table 1.

κ_2	γ	d_h
4.0	-0.30(1)	3.95 (5)
4.5	-0.12(1)	3.57 (3)

Table 1: Measurements of γ and d_h for the DAGM model. $n_g = 3$ and $N_4 = 4000$.

Our measurements were made on the $N_4 = 4000$ system with $n_g = 3$. It was therefore possible to measure only d_h (and not d_H) by using standard techniques [13, 15, 14]. The number $n_V(r)$ of 4–simplices at distance r from a given 4–simplex was measured and a fit was made to the function $C(r + a)^{d_h - 1}$ where a is the so–called "shift". r is the "4–simplex distance" which is a measure of the geodesic distance between 4–simplices. The use of the shift permits us to estimate d_h even for this moderate size of the system as can be seen in fig. 2. A slight modification of the shift a results in a much poorer fit for small r.

However, the important point is that for $\alpha = 1$ and *without* the measure term one moves directly from the crumpled phase and into a branched polymer phase. There is no trace of any crinkled phase and the interaction between the gauge fields and geometry is as weak as the interaction between scalar fields and geometry observed in [16], provided that the gauge fields are coupled to simplicial gravity in a way consistent with the way the scalar fields were coupled to the geometry in [16], as discussed in detail in the beginning of sec. 2.

In table 2 we present the results of the computer simulations for the number of gauge copies $n_g = 3$ and 6. The value of κ_2 is chosen so large that one is well away from the crumpled phase. We studied systems with volume $N_4 = 4000, 8000$ and 16000. Our results are summarized in table 2.

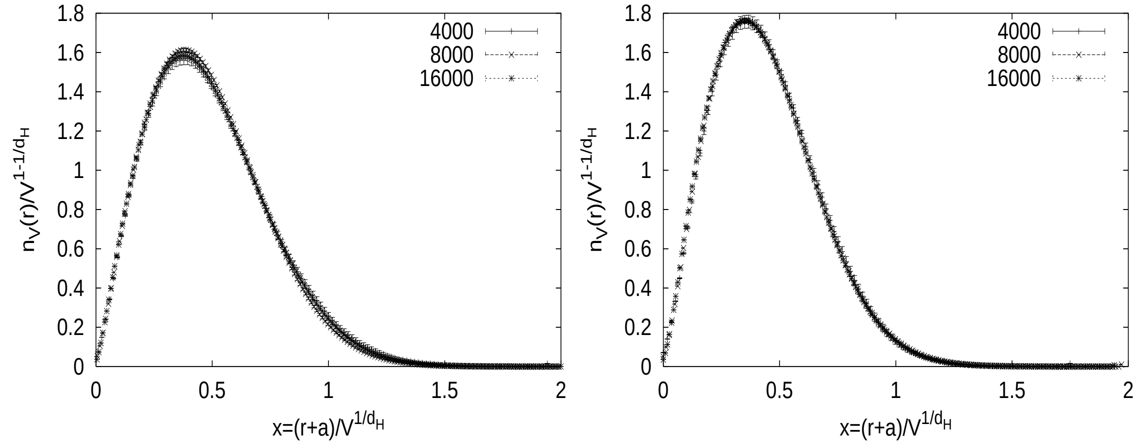

Figure 3: Calculation of d_H using finite size scaling of $n_V(r)$. The left figure corresponds to $n_g = 3$, $\kappa_2 = 4.5$, $d_H = 2.03$ and the right one to $n_g = 6$, $\kappa_2 = 7.0$, $d_H = 2.05$.

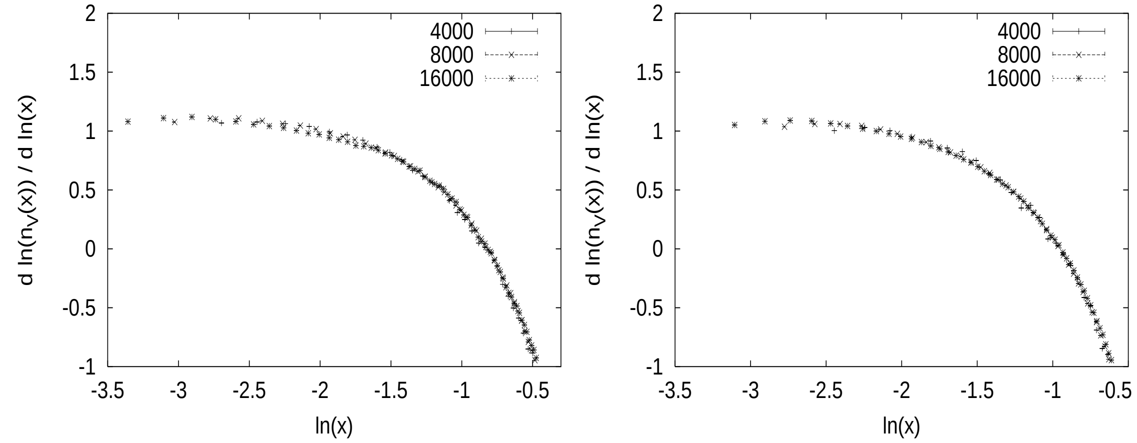

Figure 4: Calculation of d_h for $n_g = 3$, $\kappa_2 = 4.5$ (left) and $n_g = 6$, $\kappa_2 = 7.0$ (right). Here we use $x = (r + a)/V^{1/d_h}$. The shifts a are -0.5 and -1.5 respectively.

n_g	N_4	κ_2	γ	d_h	d_H
3	4000	4.5	0.49 (1)	2.1 (2)	2.03(7)
	8000	4.5	0.51(1)		
	16000	4.5	0.483(4)		
6	4000	7.0	0.44 (1)	2.2 (2)	2.05(6)
	8000	7.0	0.47 (2)		
	16000	7.0	0.470(5)		

Table 2: Measurements of γ, d_h and d_H when α=1

In fig. 3 and fig. 4 we show the computation of d_H and d_h from the scaling properties of $n_V(r)$. d_H is computed as described in [15, 14] using the relation $n_V(r) = V^{1-1/d_H} F_1((r+a)/V^{1/d_H})$. We determine d_H and a from the optimal "collapsing" of the $n_V(r)$ distributions. d_h is obtained as for the DAGM model described above. We observe that the expected scaling holds very well and that the values for d_h and d_H are consistent with being the same.

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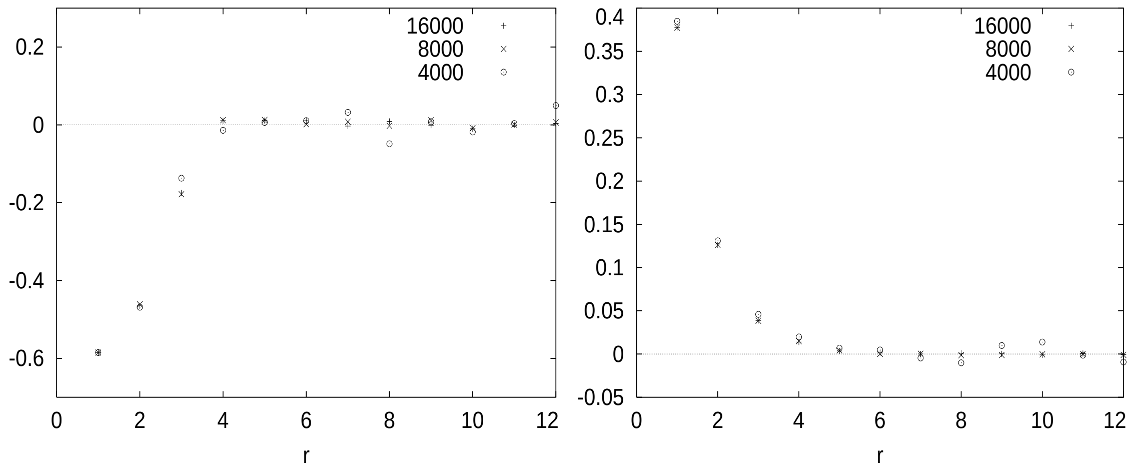

Figure 5: The connected curvature-curvature (left) and action-action (right) correlation functions. $n_g = 6$ and $k_2 = 7.0$. The $n_g = 3$ plots are falling on top of the $n_g = 6$.

We have also measured the connected part of curvature–curvature and action–action correlators (we refer to [17] for the details). One expects for two observables A_i, $i = 1, 2$ that

$$G^{1A_i}(r) = \langle A_i \rangle G^{11}(r + \delta_{A_i})\,, \tag{4.3}$$

and that the connected part of the correlator is given by

$$\begin{aligned} G_c^{A_1A_2}(r) \equiv G^{A_1A_2}(r) - \langle A_1 \rangle G^{1A_2}(r + \delta_{A_1}) - \langle A_2 \rangle G^{1A_1}(r + \delta_{A_2}) \\ + \langle A_1 \rangle \langle A_2 \rangle G^{11}(r + \delta_{A_1} + \delta_{A_2})\,. \end{aligned} \tag{4.4}$$

In the above formulas "1" refers to the unity operator and $G^{11}(r) \equiv n_V(r)$. In the case of curvature–curvature correlator we determine the best $\langle A_i \rangle$ and δ_i by collapsing $G^{1A_i}(r)$ with $G^{11}(r)$. We use these values in (4.4) in order to produce the plot in fig. 5. Similarly we proceed for the action–action correlator. Observe that curvature–curvature fluctuations are short range and independent of the system size. The same is true for action–action correlators which fall off at the same scale as the curvature–curvature ones. Identical plots are obtained for the AGM model so we conclude that the measure factor does not smoothen the configurations enough for the fields to fluctuate at large scales. The observed behaviour means that using the scaled distances the correlations become ultra-local. Similar pattern was observed in the crinkled and crumpled phases. This remains in the qualitative agreement with the curvature-curvature correlations measured in pure gravity [18], where the definition of the connected correlator was different.

The value for γ as well as those of d_h and d_H are consistent with those of ordinary branched polymers. The branched polymer interpretation is also corroborated by the fact that $\langle N_0 \rangle / N_4 \approx 1/4$ which is the upper kinematic bound expected from branched polymers, plus the fact that we see no vertices of high order.

4.2 The case $\alpha \neq 1$

With the negative results of the above subsection in mind we turn to the more general action (2.16) with $\alpha \neq 1$ in a search for new and potentially interesting fixed points from the point of view of continuum physics. By performing the duality transformation for a general α we obtain an action

$$\sum_t \left(\sum_{k=1}^{\nu_g} o^{\alpha}(t) p^2(k,t) \right) + \frac{\alpha n_g}{2} \ln o(t) \,. \tag{4.5}$$

Thus there is a possibility that a choice of negative α might bring us to a phase resembling or being identical to the crinkled phase, since the negative weight factor will imitate the situation encountered in the AGM model. We show the result of the measurements for the number of gauge fields $n_g = 3$ and $\alpha = -0.5$ and -1. The value of κ_2 is chosen such that we avoid the crumpled phase.

The results of the measurements of γ and d_h is shown in table 3. There is a clear tendency for the values to γ and d_h to drift from the branched polymer values toward the crinkled values, in accordance with expectations. Some limited statistics that we obtained for the $\alpha = -1.5$ model suggest that γ becomes even more negative. We have taken this as evidence that we see not really new physics with the modified weights, but rather an effective change in measure, more or less in the same way as the original AGM model differs from our model by measure term.

α	γ	d_h
-0.5	0.30(2)	2.65 (3)
-1.0	-0.51(1)	3.5 (2)

Table 3: Measurements of γ and d_h for $\alpha < 0$. $n_g = 3$, $N_4 = 4000$ and $\kappa_2 = 4.5$.

5. Discussion

Gauge fields contribute far more to the conformal anomaly than scalar fields. Thus they are expected to play an important role in effective models of four-dimensional quantum gravity where the (infrared) dynamics is dictated by the conformal anomaly [7]. The conjectured scaling behavior of these model are quite similar to the observed (pseudo) scaling behavior observed in four-dimensional simplicial quantum gravity, and this led to the expectation that gauge fields might couple stronger to geometry than the scalar fields used so far. So far, computer simulations have revealed only a weak coupling between the scalar fields and the geometry, and the presence of scalar fields have not led to any quantitative change in the phase diagram of simplicial quantum gravity.

The first Monte Carlo simulations of simplicial quantum gravity coupled to gauge fields seemed to be in accordance with the above philosophy. Indeed, for the first time one observed a genuine back-reaction of the matter fields on the quantum geometry. However, as already remarked in the original article [5] and verified in detail later [6],

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major part of the interaction between geometry and the matter could be accounted for by the term

$$S_{eff} = \frac{n_g}{2} \sum_t \ln(o(t)) \,, \tag{5.1}$$

where $o(t)$ denotes the order of the triangle t. This term is an ultra-local measure term and it is (as already remarked in [5]) unlikely that it contributes to the infrared dynamics related to the conformal anomaly in the model proposed in [7, 8]. However, it still left us with the puzzle why gauge fields seemingly interact so much stronger with geometry than ordinary scalar fields.

In the work presented here we have verified the original results obtained in the AGM model by an independent Monte Carlo simulation working with gauge fields on the lattice dual to that of the of the AGM model. In addition we have shown that there is *no* major difference between the interaction of scalar fields with geometry and the interaction of gauge fields with geometry. In fact, if the gauge fields are coupled to geometry in a way consistent with the way the scalar fields were coupled to geometry, the gauge and the scalar fields have similar weak coupling to the geometry. We have explained how the weight factor (5.1) arose in the transformation from gauge fields living on the dual links of a triangulation to gauge fields living on the links themselves.

Turning the arguments around, our results lead to the prediction that scalar fields living on the vertices of four-dimensional triangulations will couple as strongly to the geometry as the gauge fields in the AGM model.

We extended that gauge field model coupled to geometry by modifying the weight given to each plaquette, in order to explore further the phase-space structure of matter coupled to geometry. However, the only new phase we could identify is presumably identical to the crinkled phase already found in the AGM model. The weight generated by the transformation from the dual links to the ordinary links of a triangulation gave support to this interpretation.

A. Appendix

All moves require modification of some plaquettes by introducing new dual links into plaquettes or by destroying the dual links. Finding probabilities for the moves resembles the problem of the gaussian scalar fields, however the gauge invariance of the action requires some important changes to maintain the detailed balance and at the same time to keep the acceptance reasonable. In this section we shall discuss the case of one gauge potential and the parameter $\alpha = 1$.

Let us first describe the process of introducing new degrees of freedom. For simplicity let us discuss only the situation of the move, where a new vertex is added in the center of a simplex. There are now 10 new gauge potentials. We consider only the part of the gauge action involved in the move. Before the move is attempted we have

$$\exp(-S_A(p_i)) = \exp\left(-\sum_i^{10} \beta_i p_i^2\right), \tag{A.1}$$

where $\beta_i = 1/o(t_i)$ and p_i are the *external plaquette* variables. The modified action becomes

$$\exp(-S_B(p_i, x_i)) = \exp\left(-\sum_i^{10} \beta_i'(p_i + x_i)^2 - \beta' \sum_{i,j} x_i M_{ij} x_j\right), \tag{A.2}$$

where x_i are new fields, β_i' are the new weights, typically $\beta_i' = 1/(o(t_i)+1)$, the matrix M_{ij} contains the geometric information about ten new *internal plaquettes* formed in the move. These new plaquettes Π_k can be written as

$$\begin{aligned} \Pi_k &= \sum U_{kj} x_j \\ M_{ij} &= \sum_k U_{ki} U_{kj}\,, \end{aligned} \tag{A.3}$$

where U_{ki} are ± 1 or zero reflecting the orientation of the x_i fields. The internal plaquettes have always the order three, so $\beta' = 1/3$ typically. When performing the move we shall have to

- decide whether we make the move or not and
- if yes, generate the new fields with the distribution resulting from (A.2).

Let us define

$$\begin{aligned} \mathcal{N}_B(p_i) &= \int \prod dx_i \exp(-S_B(p_i, x_i)) \\ &= \sqrt{\frac{\pi^{10}}{\det(\mathbf{Q})}} \exp\left(-\bar{S}_B(p_i)\right), \\ Q_{ij} &= \beta' M_{ij} + \beta_i' \delta_{ij}\,, \end{aligned} \tag{A.4}$$

where $\bar{S}_B(p_i)$ is the *global minimum* of $S_B(p_i, x_i)$ with respect to x_i. This minimum can of course be explicitly calculated in terms of p_i and the matrix $\mathbf{Q}^{-1}$:

$$\bar{S}_B(p_i) = \sum_i \beta_i' p_i^2 - \sum_{ij} p_i Q_{ij}^{-1} p_j\,. \tag{A.5}$$

The quantity $w_B(p_i, x_i) = \exp(-S_B(p_i, x_i))/\mathcal{N}_B(p_i)$ is a normalized gaussian distribution of the x_i fields. Let $\mathcal{P}_B = \pi_B \mathcal{N}_B$ be the probability that the move will be performed and, if accepted, the new fields be generated with this distribution. It is easy to check that the transition probability $P(A \to B)$ satisfies

$$\begin{aligned}\exp(-S_A)P(A \to B) &= \exp(-S_A)\mathcal{P}_B w_B(p_i, x_i) \\ &= \pi_B \exp(-S_A - S_B)\,. \end{aligned} \tag{A.6}$$

The form (A.6) will be useful in order to discuss the detailed balance condition. The important point is that the only non-trivial quantity, $\mathcal{N}_B(p_i)$ can be calculated also in the new configuration $\{P_i = p_i + x_i, \Pi_k\}$. Using (A.4) we can express $\bar{S}_B$ as a *global minimum* of $\sum_i \beta_i'(P_i + y_i)^2 + \beta' \sum_k (\Pi_k + \sum_j U_{kj} y_j)^2$ with respect to the shifted variables y_i. The only difference is the shift of variables and the value of the minimum is unchanged. This property is very important, because we shall have to calculate $\mathcal{N}_B$ both for the move and it's inverse.

Let us now discuss the inverse move: we destroy 10 internal plaquettes Π_k and modify values of 10 external plaquettes, eliminating 10 gauge fields produced before. The naive proposition would be to store the gauge fields x_i and simply to delete them. The gauge fields however have no gauge invariant values so the transition performed this way would depend on the particular gauge choice - in fact even if plaquette variables are small (because of the gaussian weights) the gauge fields may become large and in effect the transition may be blocked completely.

In the program we use a different approach. Rather than storing the gauge fields we try to reconstruct them when needed. When trying to delete fields x_i we have first to decide what gauge choice we make. In other words we shall study the change $P_i \to p_i = P_i - x_i$ where x_i satisfy the set of gauge–invariant constraints $\sum U_{ki} x_i = \Pi_k$. Notice that not all these constraints are independent because of the Bianchi identities between the ten plaquettes Π_k. Only six of them have to be used, reducing the number of independent degrees of freedom to four. As before let us consider only the part of the action engaged in the move. Before the move is attempted we have

$$\exp(-S_B(P_i, \Pi_k)) = \exp\left(-\sum_i^{10} \beta_i'\, P_i^2 - \beta' \sum_k^{10} \Pi_k^2\right) \tag{A.7}$$

After the move we have:

$$\exp(-S_A(P_i, x_i)) = \exp\left(-\sum_i^{10} \beta_i (P_i - x_i)^2\right), \tag{A.8}$$

As before we shall have to

- decide if we perform the move and
- if yes, perform it, i.e. choose some values of x_i following from (A.8).

Let me define as before:

$$\begin{aligned}\mathcal{N}_A(P_i,\Pi_k) &= \int \prod_i^{10} dx_i \prod_k^{6} \delta\left(\Pi_k - \sum U_{kj}x_j\right) \exp(-S_A(P_i,x_i)),\\ &= \sqrt{\frac{\pi^4}{\det\mathbf{N}}}\exp(-\bar{S}_A(P_i,\Pi_k)),\end{aligned} \tag{A.9}$$

where the matrix $\mathbf{N}$ depends only on β_i and $\bar{S}_A(P_i,\Pi_k)$ is the *conditional minimum* of $S_A(P_i,x_i)$ with gauge–invariant set of conditions imposed on the x_i fields. The integration in (A.9) can be viewed as integration over all possible gauge choices for the fields x_i. It is simple to give the explicit formula for $\bar{S}_A$ in terms of P_i, Π_k and the matrix $\mathbf{N}^{-1}$. We shall not write it here. Let us however note that $\mathcal{N}_A(P_i,\Pi_k)$ can be also calculated in the new configuration $\{P_i,\Pi_k\} \to \{p_i, 0\}$ using a simple relation $\mathcal{N}_A(P_i,\Pi_k) = \mathcal{N}_A(p_i,0)$.

The quantity $w_A(P_i,\Pi_k,x_i) = \exp(-S_A(P_i,x_i)/\mathcal{N}_A(P_i,\Pi_k))$ is a normalized probability for x_i satisfying the gauge-invariant constraints. As before let us choose the probability $\mathcal{P}_A = \pi_A\mathcal{N}_A(P_i,\Pi_k)$ to be the probability of performing the move. If accepted we generate four independent x_i from the distribution w_A. The remaining six can be calculated using the constraints. We can check that the transition probability $P(B \to A)$ satisfies

$$\exp(-S_B)P(B\to A) = \pi_A \exp(-S_A - S_B). \tag{A.10}$$

Equations (A.6) and (A.10) can be used to write the detailed balance condition:

$$\exp(-S_A)P(A\to B) = \exp(-S_B)P(B\to A)$$

meaning

$$\frac{\pi_B}{\pi_A} = 1\,, \qquad \text{or} \qquad \frac{\mathcal{P}_B}{\mathcal{P}_A} = \frac{\mathcal{N}_B}{\mathcal{N}_A}\,. \tag{A.11}$$

As usual, this form of the detailed balance condition suggests that in order to maximize the acceptance we need to know both $\mathcal{N}_B$ and $\mathcal{N}_A$.The generalization to the case of more gauge fields and $\alpha \neq 1$ is trivial.

The same method can be used to calculate the weights for other moves. The general rule is that we need to consider at the same time both the move and it's inverse.

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